Z Sieve Method

Abstract

A primary sieve method, Z Sieve Method, is presented. With this method, formulas are deduced for the estimation of the number of twins and the number of Goldbach partitions, with accuracies comparable with that of Hardy-Littlewood's and Grandall-Pomenrance's.

Based on this method, the nature of $\prod_{P\geq 3}\frac{P(P-2)}{(P-1)^2}$, $\prod_{P|N}\frac{P-1}{P-2}$, **and other terms of formulas are expounded.**

Furthermore, this method shows the probable of the formulas for calculation of the number of twins and the number of Goldbach partitions, for which the results are exactly the same as real values and of birth of the proofs of these two conjectures.

In this paper, the main purpose of the Z Sieve Method, is to apply it to Twin Conjecture and Goldbach Conjecture.

Although Prime Theorem is well known, for the convenience of expressing and applying Z Sieve Method, I apply it to express Prime Theorem as below.

For an odd number series: 1, 3, 5…N-1 (N is an even number), denote S(i) is the number of the terms, which is sifted by P_i only and we have

$$S(1)=\left[\frac{N+P_1}{2P_1}\right];$$

$$S(2)=\left[\frac{N+P_2}{2P_2}\right]-\left[\frac{N+P_1P_2}{2P_1P_2}\right];$$

$$S(3)=\left[\frac{N+P_3}{2P_3}\right]-\left[\frac{N+P_1P_3}{2P_1P_3}\right]-\left[\frac{N+P_2P_3}{2P_2P_3}\right]+\left[\frac{N+P_1P_2P_3}{2P_1P_2P_3}\right];$$

$$S(4)=\left[\frac{N+P_4}{2P_4}\right]-\sum_{1\leq i<4}\left[\frac{N+P_iP_4}{2P_iP_4}\right]+\sum_{1\leq i<j<4}\left[\frac{N+P_iP_jP_4}{2P_iP_jP_4}\right]-\left[\frac{N+P_1P_2P_3P_4}{2P_1P_2P_3P_4}\right];$$

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$$S(r)=\left[\frac{N+P_r}{2P_r}\right]-\sum_{1\leq i<r}\left[\frac{N+P_iP_r}{2P_iP_r}\right]+\sum_{1\leq i<j<4}\left[\frac{N+P_iP_jP_r}{2P_iP_jP_r}\right]-...+(-1)^{r+1}\left[\frac{N+P_1P_2...P_r}{2P_1P_2...P_r}\right],$$ wher

e P is a prime and $3 = P_1 < P_2 ... < P_r \le \sqrt{N} < P_{r+1}$. The number of the total terms is $\left[\frac{N}{2}\right]$ or $\left[\frac{N+1}{2}\right]$. Therefore, the number of primes less N, $\pi(N)$ is:

$$\pi(N) = \left[\frac{N+1}{2}\right] - \sum_{1\le i\le r}\left[\frac{N+P_i}{2P_i}\right] + \sum_{1\le i<j\le r}\left[\frac{N+P_iP_j}{2P_iP_j}\right] - \sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k}{2P_iP_jP_k}\right] +$$

$$+ ... + (-1)^r\left[\frac{N+P_1P_2...P_r}{2P_1P_2...P_r}\right] + r.$$

To figure the number of twin primes less N, $Z_2(N)$, by Z Sieve Method make 2 odd number series A and B and one composite series C (A,B) as below:

A: 1, 3, 5... N-5, N-3, N-1;

B: 3, 5, 7... N-3, N-1; N+1;

C: (1, 3), (3, 5), (5, 7)...(N-5, N-3), (N-3, N-1), (N-1, N+1),

where $A_i + 2 = B_i$ and C_i (A_i, B_i).

Count C_i is a composite term, if either A_i or B_i is a composite. C_i is a twin term, if both A_i and B_i is a prime. After sifting all the composite terms from C series, the rest of the terms of C series are twins except C_1 (1, 3).

For sifting series C, do the way as S(i) above and follow the steps below:

1-A Sift all the composite terms by P_1 from series C(A): AS(1);

1-B Sift all the composite terms by P_1 from series C(B): BS(1);

2-A Sift all the composite terms by P_2 only from the rest terms of series C(A): AS(2);

2-B Sift all the composite terms by P_2 only from the rest terms of series C(B): BS(2);

3-A Sift all the composite terms by P_3 only from the rest terms of series C(A): AS(3);

3-B Sift all the composite terms by P_3 only from the rest terms of series C(B): BS(3);

...

r-A Sift all the composite terms by P_r only from the rest terms of series C(A): AS(r);

r-B Sift all the composite terms by P_r only from the rest terms of series C(B): BS(r).

The number of the total terms of C series is $\left[\frac{N}{2}\right]$ or $\left[\frac{N+1}{2}\right]$. Therefore,

$Z_2(N) = \left[\frac{N+1}{2}\right]$ - AS(1) – BS(1) – AS(2) – BS(2) – AS(3) – BS(3) -... - AS(r) – - BS(r) + $T_2(\sqrt{N})$ + F(0/-1).

Because $A_i + 2 = B_i$, if $P_1 | A_i$, P_1 is not divisible by B_i; therefore $AS(1) = \left[\frac{N+P_1}{2P_1}\right]$ and $BS(1) = \left[\frac{N+P_1}{2P_1}\right]$. AS(2) is the number of the composite terms divisible by P_2 on series C (A) minus the number of the composite terms divisible by P_1P_2 on series C (A) and minus the number of the composite terms on series C (B), which is $P_2 | A_i$ and $P_1 | B_i$

$$AS(2)=\left[\frac{N+P_2}{2P_2}\right]-\left[\frac{N+P_1P_2}{2P_1P_2}\right]-\left[\frac{N+P_1P_2+V(2)_{1A}P_2}{2P_1P_2}\right],$$

where $V(2)_{1A}$ stands for a factor related to the sifting composites of P_2 on C(A) series involving total 2 primes (P_1 and P_2) and the positions of the composites of P_1 on C(B) series. $V(2)_{1A}$ is determined by their positions of the corresponding composites of P_2 and P_1 on C(A) and C(B).

Similar to AS(2),

$$BS(2)=\left[\frac{N+P_2}{2P_2}\right]-\left[\frac{N+P_1P_2}{2P_1P_2}\right]-\left[\frac{N+P_1P_2+V(2)_{1B}P_2}{2P_1P_2}\right];$$

$$AS(3)=\left[\frac{N+P_3}{2P_3}\right]-\left[\frac{N+P_1P_3}{2P_1P_3}\right]-\left[\frac{N+P_1P_3+V(2)_{1A}P_3}{2P_1P_3}\right]-\left[\frac{N+P_2P_3}{2P_2P_3}\right]-$$
$$-\left[\frac{N+P_2P_3+V(2)_{2A}P_3}{2P_2P_3}\right]+\left[\frac{N+P_1P_2P_3}{2P_1P_2P_3}\right]+\left[\frac{N+P_1P_2P_3+V(3)_{2A}P_1P_3}{2P_1P_2P_3}\right]+$$
$$+\left[\frac{N+P_1P_2P_3+V(3)_{1A}P_2P_3}{2P_1P_2P_3}\right]+\left[\frac{N+P_1P_2P_3+V(3)_{1,2A}P_3}{2P_1P_2P_3}\right];$$

$$BS(3)=\left[\frac{N+P_3}{2P_3}\right]-\left[\frac{N+P_1P_3}{2P_1P_3}\right]-\left[\frac{N+P_1P_3+V(2)_{1B}P_3}{2P_1P_3}\right]-\left[\frac{N+P_2P_3}{2P_2P_3}\right]-$$
$$-\left[\frac{N+P_2P_3+V(2)_{2B}P_3}{2P_2P_3}\right]+\left[\frac{N+P_1P_2P_3}{2P_1P_2P_3}\right]+\left[\frac{N+P_1P_2P_3+V(3)_{2B}P_1P_3}{2P_1P_2P_3}\right]+$$
$$+\left[\frac{N+P_1P_2P_3+V(3)_{1B}P_2P_3}{2P_1P_2P_3}\right]+\left[\frac{N+P_1P_2P_3+V(3)_{1,2B}P_3}{2P_1P_2P_3}\right];$$

$$AS(r)=\left[\frac{N+P_r}{2P_r}\right]-\sum_{1\le i<r}\left[\frac{N+P_iP_r}{2P_iP_r}\right]-\sum_{1\le i<r}\left[\frac{N+P_iP_r+V(r)_{iA}P_r}{2P_iP_r}\right]+$$
$$+\sum_{1\le i<j<r}\left[\frac{N+P_iP_jP_r}{2P_iP_jP_r}\right]+\sum_{1\le i<j<r}\left[\frac{N+P_iP_jP_r+V(r)_{iA}P_jP_r}{2P_iP_jP_r}\right]+$$
$$+\sum_{1\le i<j<r}\left[\frac{N+P_iP_jP_r+V(r)_{i,jA}P_r}{2P_iP_jP_r}\right]+\ldots+(-1)^r\left[\frac{N+P_1P_2\ldots P_r}{2P_1P_2\ldots P_r}\right]+$$
$$+(-1)^r\sum_{1\le i<r}\left[\frac{N+P_1P_2\ldots P_r+V(r)_{iA}P_j\ldots P_r}{2P_1P_2\ldots P_r}\right]+$$
$$+\ldots+(-1)^r\left[\frac{N+P_1P_2\ldots P_r+V(r)_{i,j\ldots r-1A}P_r}{2P_1P_2\ldots P_r}\right];$$

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$$BS(r)=\left[\frac{N+P_r}{2P_r}\right]-\sum_{1\le i<r}\left[\frac{N+P_iP_r}{2P_iP_r}\right]-\sum_{1\le i<r}\left[\frac{N+P_iP_r+V(r)_{iB}P_r}{2P_iP_r}\right]+$$

$$+\sum_{1\le i<j<r}\left[\frac{N+P_iP_jP_r}{2P_iP_jP_r}\right]+\sum_{1\le i<j<r}\left[\frac{N+P_iP_jP_r+V(r)_{iB}P_jP_r}{2P_iP_jP_r}\right]+$$

$$+\sum_{1\le i<j<r}\left[\frac{N+P_iP_jP_r+V(r)_{i,jB}P_r}{2P_iP_jP_r}\right]+\ldots+(-1)^r\left[\frac{N+P_1P_2\ldots P_r}{2P_1P_2\ldots P_r}\right]+$$

$$+(-1)^r\sum_{1\le i<r}\left[\frac{N+P_1P_2\ldots P_r+V(r)_{iB}P_j\ldots P_r}{2P_1P_2\ldots P_r}\right]+$$

$$+\ldots+(-1)^r\left[\frac{N+P_1P_2\ldots P_r+V(r)_{i,j\ldots r-1B}P_r}{2P_1P_2\ldots P_r}\right].$$

$Z_2(N)=\left[\frac{N+1}{2}\right]$ - AS(1) – BS(1) – AS(2) – BS(2) – AS(3) – BS(3) -... - AS(r) – BS(r) + $T_2(\sqrt{N})$ + F(0/-1)=

$$=\left[\frac{N+1}{2}\right]-2\sum_{1\le i\le r}\left[\frac{N+P_i}{2P_i}\right]+2\sum_{1\le i<j\le r}\left[\frac{N+P_iP_j}{2P_iP_j}\right]+$$

$$+\sum_{1\le i<j\le r}\left[\frac{N+P_iP_j+V(2)_{iA}P_j}{2P_iP_j}\right]+\sum_{1\le i<j\le r}\left[\frac{N+P_iP_j+V(2)_{iB}P_j}{2P_iP_j}\right]-$$

$$-2\sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k}{2P_iP_jP_k}\right]-$$

$$-\sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k+V(3)_{iA}P_jP_k}{2P_iP_jP_k}\right]-\sum_{1\le i<j\le r}\left[\frac{N+P_iP_jP_k+V(3)_{iB}P_jP_k}{2P_iP_jP_k}\right]-$$

$$-\sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k+V(3)_{i,jA}P_k}{2P_iP_jP_k}\right]-\sum_{1\le i<j\le r}\left[\frac{N+P_iP_jP_k+V(3)_{i,jB}P_k}{2P_iP_jP_k}\right]+$$

+...+

$$+(-1)^r 2\left[\frac{N+P_iP_j...P_r}{2P_iP_j...P_r}\right]+(-1)^r\sum_{1\le i<r}\left[\frac{N+P_iP_j...P_r+V(r)_{iA}P_jP_k...P_r}{2P_iP_j...P_r}\right]+$$

$$+(-1)^r\sum_{1\le i,j<r}\left[\frac{N+P_iP_j...P_r+V(r)_{ijA}P_k...P_r}{2P_iP_j...P_r}\right]+$$

$$+...+(-1)^r\left[\frac{N+P_iP_j...P_r+V(r)_{ij...(r-1)A}P_r}{2P_iP_j...P_r}\right]+$$

$$+(-1)^r\sum_{1\le i<r}\left[\frac{N+P_iP_j...P_r+V(r)_{iB}P_jP_k...P_r}{2P_iP_j...P_r}\right]+$$

$$+(-1)^r\sum_{1\le i,j<r}\left[\frac{N+P_iP_j...P_r+V(r)_{ijB}P_k...P_r}{2P_iP_j...P_r}\right]+$$

$$+...+(-1)^r\left[\frac{N+P_iP_j...P_r+V(r)_{ij...(r-1)B}P_r}{2P_iP_j...P_r}\right]+T_2(\sqrt{N})+F(0/-1),$$

where $T_2(\sqrt{N})$ is the number of the real twin primes less $\sqrt{N}$, which should be added due to all the primes less $\sqrt{N}$ sure including twins less $\sqrt{N}$ were sifted, and F(0/-1) is the adjusted term related to the first term C(1,3) and the last term C(N-1, N+1). F(0/-1) = -1, in 3 cases: first, N < 9; second, (N-1) is a prime and (N+1) is a composite; third, both (N-1) and (N+1) are a composite and the smallest factor of (N+1) is smaller than that of (N-1). Otherwise, F(0/-1) = 0.

In discussing factor V, denote common formats: P_a, P_b, $V(a+b)_{aA}$, and $\left[\frac{N+P_aP_b+V(a+b)_{aA}P_b}{2P_aP_b}\right]$, where P_a and P_b are a group of primes and $V(a+b)_{aA}$, stands for a factor related to the sifting composites of P_b on C(A) series involving total a+b primes (P_a and P_b) and composites of P_a on C(B) series.

For an odd number system on series C(A):
P_b, $3P_b$, $5P_b$, ... $(2P_a-1)P_b$, each of which is divisible by P_b, and the number of the total terms of the number system is P_a. Therefore, the corresponding number system on series C(B) is a complete residue system (mod P_a):
P_b+2, $3P_b+2$, $5P_b+2$, ... $(2P_a-1)P_b+2$. This means there is one and only one term $C_i(A_i, B_i)$ to meet the condition: $P_b|A_i$ and $P_a|B_j$.

There are 2 extreme values of $V(a+b)_{aA}$: P_a-1 and $1-P_a$, when $N = A_i = P_b$ and $N = A_i = (2P_a-1)P_b$ respectively. The range of the value of $\frac{P_aP_b+V(a+b)_{aA}P_b}{2P_aP_b}$ is fractional: $\frac{1}{2P_a}\le\frac{P_aP_b+V(a+b)_{aA}P_b}{2P_aP_b}\le 1-\frac{1}{2P_a}$ or $\left|\frac{P_aP_b+V(a+b)_{aA}P_b}{2P_aP_b}\right|<1$.

In addition, denote an odd integer m $(1 \le m \le 2P_a - 1)$ to have mP_b on series C(A), where $P_b > P_a$, and to have $\frac{mP_b + P_aP_b + V(a+b)_{aA} P_b}{2P_aP_b} = 1$ and $P_a | mP_b + 2$ for sifting composite terms of P_b on series C(A) and composite terms of P_a on series C(B). Likewise, for an odd number system P_b, $3P_b$, $5P_b$ ... $(2P_a-1)P_b$ on series C(B) and its corresponding system P_b-2, $3P_b$-2, $5P_b$-2... $(2P_a-1)P_b$-2 on series C(A), denote an positive integer m' (1< m' <$2P_a$-1), where $P_b > P_a$, to have $\frac{m'P_b + P_aP_b + V(a+b)_{aB} P_b}{2P_aP_b} = 1$ and $P_a | m'P_b - 2$ for sifting composite terms of P_b on series C(B) and composite terms of P_a on series C(A).

From $\frac{mP_b + P_aP_b + V(a+b)_{aA} P_b}{2P_aP_b} = 1$ and $\frac{m'P_b + P_aP_b + V(a+b)_{aB} P_b}{2P_aP_b} = 1$, we have $V(a+b)_{aA} = P_a - m$ and $V(a+b)_{aB} = P_a - m'$. From $P_a | mP_b + 2$ and $P_a | m'P_b - 2$, we have $P_a | m + m'$. Since 1< m, m' <$2P_a$-1, one solution is m=m'=P_a, then $V(a+b)_{aA} = P_a - m = 0$ and $V(a+b)_{aB} = P_a - m' = 0$. We may write $V(a+b)_{aA} = -V(a+b)_{aB}$.

If both m and m' is greater than P_a, $2P_a$ < m+m' < $4P_a$ – 2 because 1< m, m' <$2P_a$-1, there is only one number, $3P_a$, between $2P_a$ and $4P_a$ – 2, which can be divided by P_a. However, both m and m' are an odd number, so the sum of m and m' is an even number then $m + m' \ne 3P_a$. For the same reason, if both m and m' is smaller than P_a, the sum of m and m' can not be divided by P_a either.

When m > P_a and m' < P_a or conversely and if m = P_a + n and m' = P_a – n, then $P_a | m + m' = 2P_a$. In this case,
$V(a+b)_{aA} = P_a - m = P_a - (P_a + n) = -(P_a - m') = -V(a+b)_{aB}$.
Since $V(a+b)_{aA} = -V(a+b)_{aB}$ for all situations, we merge $V(a+b)_{aA}$ and $V(a+b)_{aB}$ into one term of $\pm V(a+b)_a$. Then,

$$Z_2(N) = \left[\frac{N+1}{2}\right] - 2\sum_{1\le i\le r}\left[\frac{N+P_i}{2P_i}\right] + 2\sum_{1\le i<j\le r}\left[\frac{N+P_iP_j}{2P_iP_j}\right] +$$
$$+ \sum_{1\le i,j<k\le r}\left[\frac{N+P_iP_j \pm V(2)_i P_j}{2P_iP_j}\right] - 2\sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k}{2P_iP_jP_k}\right] -$$
$$- \sum_{1\le i,j<k\le r}\left[\frac{N+P_iP_jP_k \pm V(3)_i P_jP_k}{2P_iP_jP_k}\right] - \sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k \pm V(3)_{i,j} P_k}{2P_iP_jP_k}\right] +$$
$$+ \ldots +$$

$$+(-1)^r 2\left[\frac{N+P_iP_j...P_r}{2P_iP_j...P_r}\right]+(-1)^r\sum_{1\le i<r}\left[\frac{N\pm V(r)_i P_j...P_r}{2P_iP_j...P_r}\right]+$$

$$+...+$$

$$+(-1)^r\left[\frac{N+P_iP_j...P_r\pm V(r)_{ij...(r-1)}P_r}{2P_iP_j...P_r}\right]+T_2(\sqrt{N})+F(0/-1).$$

Rewrite the formula of $\pi(N)$ as:

$$\pi(N)-r=\phi_1(P)=\left[\frac{N+1}{2}\right]-\sum_{1\le i\le r}\left[\frac{N+P_i}{2P_i}\right]+$$

$$+\sum_{1\le i<j\le r}\left[\frac{N+P_iP_j}{2P_iP_j}\right]-\sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k}{2P_iP_jP_k}\right]+...+(-1)^r\left[\frac{N+P_1P_2...P_r}{2P_1P_2...P_r}\right].$$

Also, rewrite the formula of $Z_2(N)$ as:

$$Z_2(N)-T_2(\sqrt{N})-F(0/-1)=\phi_2(P)=\left[\frac{N+1}{2}\right]-2\sum_{1\le i\le r}\left[\frac{N+P_i}{2P_i}\right]+$$

$$+2\sum_{1\le i<j\le r}\left[\frac{N+P_iP_j}{2P_iP_j}\right]+\sum_{1\le i<j\le r}\left[\frac{N+P_iP_j\pm V(2)_iP_j}{2P_iP_j}\right]-$$

$$-2\sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k}{2P_iP_jP_k}\right]-\sum_{1\le i,j<k\le r}\left[\frac{N+P_iP_jP_k\pm V(3)_iP_jP_k}{2P_iP_jP_k}\right]-$$

$$-\sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k\pm V(3)_{i,j}P_k}{2P_iP_jP_k}\right]+$$

$$+(-1)^{r+1}2\left[\frac{N+P_iP_j...P_r}{2P_iP_j...P_r}\right]+(-1)^{r+1}\sum_{1\le i<r}\left[\frac{N+P_iP_j...P_r\pm V(r)_iP_j...P_r}{2P_iP_j...P_r}\right]+$$

$$+...+(-1)^{r+1}\left[\frac{N+P_iP_j...P_r\pm V(r)_{ij...(r-1)}P_r}{2P_iP_j...P_r}\right].$$

We have

$$Z_2(N)-T_2(\sqrt{N})-F(0/-1)=\phi_2(P)=\phi_2(P)\times\frac{(\pi(N)-r)^2}{\phi_1(P)^2}.$$

After canceling [ ], we may have

$$\pi(N)-r=\phi_1(P)\approx\frac{N}{2}-\sum_{1\le i\le r}\frac{N}{2P_i}+\sum_{1\le i<j\le r}\frac{N}{2P_iP_j}-\sum_{1\le i<j<k\le r}\frac{N}{2P_iP_jP_k}+$$

$$+\ldots+(-1)^r\frac{N_r}{2P_1P_2\ldots P_r}=$$

$$=\frac{N}{2}\prod_{1\le i\le r}(1-\frac{1}{P_i}).$$

$$Z_2(N)-T_2(\sqrt{N})-F(0/-1)=\phi_2(P)\approx\frac{N}{2}-2\sum_{1\le i\le r}(\frac{N}{2P_i})+2^2\sum_{1\le i<j\le r}(\frac{N}{2P_iP_j})-$$

$$-2^3\sum_{1\le i<j<k\le r}(\frac{N}{2P_iP_jP_k})+\ldots+(-1)^r2^r(\frac{N}{2P_iP_j\ldots P_r})=$$

$$=\frac{N}{2}\prod_{1\le i\le r}(1-\frac{2}{P_i}).$$

$$\frac{\phi_2(P)}{\phi_1(P)^2}\approx\frac{N}{2}\prod_{1\le i\le r}(1-\frac{2}{P_i})\div(\frac{N}{2}\prod_{1\le i\le r}(1-\frac{1}{P_i}))^2=\frac{2}{N}\prod_{1\le i\le r}(1-\frac{1}{(P_i-1)^2})\approx\frac{2C_2}{N},$$

where C_2 is the famous twin constant.

$$Z_2(N)-T_2(\sqrt{N})-F(0/-1)=\phi_2(P)\times\frac{(\pi(N)-r)^2}{\phi_1(P)^2}\approx\frac{2C_2}{N}\times(\pi(N)-r)^2.$$

Compared with $Z_2(N)$ and $\pi(N)$, $T_2(\sqrt{N}),F(0/-1)$ and r can be neglected for a large N. The formula can be further simplified as: $Z_2(N)\approx\frac{2C_2}{N}\pi^2(N).$

These formulas and their deductions above clearly and precisely express the direct relation between $\pi(N)$, $Z_2(N)$, and C_2.

Although some terms are neglected, the calculated results by this simple formula are still very close to real data. These results and a formula to calculate the exact number of twins were written on a separate paper.

Goldback Conjecture is very similar to Twin Conjecture. We may apply Z Sieve Method in the same way to Goldbach Conjecture as to Twin Conjecture above.

For Goldbach Conjecture, N = P + P', we build a little different A, B, and C series below.

A: 1, 3, 5... N-5, N-3, N-1;
B: N-1, N-3, N-5... 5, 3, 1;
C: (1, N-1), (3, N-3), (5, N-5)...(N-5, 5), (N-3, 3), (N-1, 1),

where $A_i + B_i = C_i = N$ and $C_i(A_i, B_i)$.

Denote $Z_{1+1}(N)$ the number of total Goldbach partitions. First considering the N, which is divisible by 2 only, then all the terms of AS(1), BS(1), AS(2), BS(2), AS(3), BS(3)... AS(r), BS(r) are the same as $Z_2(N)$ for $Z_{1+1}(N)$ except the values of V may be different. Denote U for $Z_{1+1}(N)$ instead of V for $Z_2(N)$.

For an odd number system on series C(A):
P_b, $3P_b$, ... $(2P_a-3)P_b$, $(2P_a-1)P_b$, each of which is divisible by P_b, and the number of the total terms of the number system is P_a. Therefore, the corresponding number system on series C(B): $N-P_b$, $N-3P_b$, ... $N-(2P_a-3)P_b$, $N-(2P_a-1)P_b$ is a complete residue system (mod P_a), too. There will be one and only one term in the odd number system, $C_i(A_i, B_i)$, to meet the condition: $P_b|A_i$ and $P_a|B_j$.

Likewise, there are 2 extreme values of $U(a+b)_{aA}$: P_a-1 and $1-P_a$. Like V, we have the range of the value of $\frac{P_aP_b+U(a+b)_{aA}P_b}{2P_aP_b}$ is fractional: $\frac{1}{2P_a} \le \frac{P_aP_b+U(a+b)_{aA}P_b}{2P_aP_b} \le 1-\frac{1}{2P_a}$ or $\left|\frac{P_aP_b+U(a+b)_{aA}P_b}{2P_aP_b}\right| < 1$.

Denote m and m' are odd integers and mP_b on series C(A) ($1< m <2P_a-1$), where $P_b > P_a$, to have $\frac{mP_b+P_aP_b+U(a+b)_{aA}P_b}{2P_aP_b}=1$ and $P_a|N-mP_b$ for sifting composite terms of P_b on series C(A) and composite terms of P_a on series C(B). In the same way, we have $\frac{m'P_b+P_aP_b+U(a+b)_{aB}P_b}{2P_aP_b}=1$ and $P_a|N-m'P_b$ for sifting composite terms of P_b on series C(B) and composite terms of P_a on series C(A).

From $P_a|N-mP_b$ and $P_a|N-m'P_b$, we have $P_a|m-m'$. One possible solution is $m = m' = P_a$ and like V we may have the other only possible solution is $m = m' \neq P_a$, because $1< m, m' <2P_a-1$.

Because $U(a+b)_{aA} = P_a - m$ and $U(a+b)_{aB} = P_a - m'$, $U(a+b)_{aA} = U(a+b)_{aB}$. We merge U_A and U_B as U to have

$$Z_{1+1}(N) = \left[\frac{N+1}{2}\right] - 2\sum_{1\le i\le r}\left[\frac{N+P_i}{2P_i}\right] +$$

$$+2\sum_{1\le i<j\le r}\left[\frac{N+P_iP_j}{2P_iP_j}\right] + 2\sum_{1\le i<j\le r}\left[\frac{N+P_iP_j+U(2)_iP_j}{2P_iP_j}\right] -$$

$$-2\sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k}{2P_iP_jP_k}\right]-2\sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k+U(3)_iP_jP_k}{2P_iP_jP_k}\right]-$$

$$-2\sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k+U(3)_iP_iP_k}{2P_iP_jP_k}\right]-2\sum_{1\le i<j<k\le r}\left[\frac{N+P_iP_jP_k+U(3)_{ij}P_k}{2P_iP_jP_k}\right]+$$

$$+(-1)^r2\left[\frac{N+P_iP_j...P_r}{2P_iP_j...P_r}\right]+(-1)^r2\sum_{1\le i<r}\left[\frac{N+P_iP_j...P_r+U(r)_iP_j...P_r}{2P_iP_j...P_r}\right]+$$

$$+...+$$

$$+(-1)^r2\left[\frac{N+P_iP_j...P_r+U(r)_{ij...(r-1)}P_r}{2P_iP_j...P_r}\right]+\sum_{N-P_i=P}2.$$

There is only a difference between $Z_{1+1}(N)$ and $Z_2(N)$ for N divisible by 2 only; instead of $T_2(N)$ and F(0/-1), $Z_{1+1}(N)$ consists of $\sum_{N-P_i=P}2$. Because all the P_i are to be sifted according to Z Sieve Method, when N- P_i is a prime, 2 should be added for $Z_{1+1}(N)$.

Like $Z_2(N)$, we may have

$$Z_{1+1}(N)-\sum_{N-P_i=P}2=\phi_2(P)\times\frac{(\pi(N)-r)^2}{\phi(P)^2}\approx\frac{2C_2}{N}\times(\pi(N)-r)^2$$

and simplify it to

$$Z_{1+1}(N)\approx\frac{2C_2}{N}\pi^2(N).$$

The simplified formula is the same for both $Z_{1+1}(N)$ and $Z_2(N)$. The calculated results of $Z_{1+1}(N)$ by this simple formula for N divisible by 2 only are very close to real values. These results and a formula to calculate the exact number of Goldbach partitions were written on a separate paper, where the factor, $\prod_{P_i|N}\frac{P_i-1}{P_i-2}$, for N divisible by odd numbers was deduced.